

# Density fluctuations in a quasi-one-dimensional Bose gas as observed in free expansion

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We study, within a framework of the classical fields approximation, the density correlations of a weakly interacting expanding Bose gas for the whole range of temperatures across the Bose-Einstein condensation threshold. We focus on elongated quasi-one-dimensional systems where there is a huge discrepancy between the existing theory and experimental results (A. Perrin et al., *Nature Phys.* 8, 195 (2012)). We find that the density correlation function is not reduced for temperatures below the critical one as it is predicted for the ideal gas or for a weakly interacting system within the Bogoliubov approximation. This behavior of the density correlations agrees with the above mentioned experiment with the elongated system. Although the system was much larger then studied here we believe that the behavior of the density correlation function found there is quite generic. Our theoretical studies indicate also large density fluctuations in the trap in the quasicondensate regime where only phase fluctuations were expected. We argue that the enhanced density fluctuations can originate in the presence of interactions in the system, or more precisely in the existence of spontaneous dark solitons in the elongated gas at thermal equilibrium.

Correlations are the essence of any many body systems. In particular, the quantum features of the system are manifested by some unusual correlations. The first order coherence is the basic criterion used as the definition of a Bose-Einstein condensate. And indeed, very soon after observation of trapped atomic condensates the first order coherence of such systems, manifesting itself in the ability to produce the interference fringes in a two-slit experiment, had been proven [1] experimentally. Higher order correlations leading to atom bunching were established from collisions and three-body losses [2]. Although the Glauber coherence theory introduced to characterize correlations of quantum electromagnetic field is well established now, the issue of coherence of a matter field is still under intensive investigation.

A Bose-Einstein condensate is a matter wave analogue of a coherent light. A very important question is how far this analogy can be pursued. The first difference is that atoms exist in a Fock states only: the states which are the eigenstates of the particle number operator. As a consequence, the coherent state of a matter field, understood as the exact analogue of the coherent electromagnetic field, does not exist. On the other hand, the atomic field can exhibit a higher order coherence too. The coherence ought to be understood here as the ability to produce the interference patterns not only in a single-particle detection schemes, but also in a simultaneous detection of larger number of atoms. Although the phase of a number state can be arbitrary, the Fock states can interfere in a single realization of the system [3] when no averaging over global phase is performed. Therefore higher order coherence of atomic condensates, as can be observed in a single realization of the system, is an interesting issue and in fact this is the property of the system which characterizes a genuine Bose-Einstein condensate [4]. The correlation functions of a Bose-Einstein condensate – a

coherent atomic wave – can be contrasted with the correlations of fermionic atoms and/or thermal nondegenerate clouds.

The famous experiment by R. Hanbury Brown and R.Q. Twiss [5] exploring a new type of interferometer to measure the angular diameter of radio stars initiated a modern theory of coherence. The Hanbury Brown and Twiss effect is rooted in properties of the density-density correlations or equivalently, the second order correlation function. The bunching effect for thermal photons originating from many particle interference enhancing probability of many photons detection can be contrasted to the absence of such an effect for coherent light when only a single mode is populated. The atom counting experiments with metastable bosonic or fermionic helium atoms, exploring thermal density-density correlations, show bunching of nondegenerate bosons and antibunching effects for fermions [6]. Similar results were obtained in experiments with nondegenerate gas at optical lattices [7] and atomic lasers [8]. Direct measurement of the second and the third order correlation function of Bose-Einstein condensates is an important proof of their high coherence [9].

The correlation functions of the atomic field, contrary to the electromagnetic waves, depend not only on the temperature, but also on the interactions and the dimensionality of the system. The density and phase of a three-dimensional condensate do not fluctuate. The situation is different in lower dimensions. In a two-dimensional Bose gas the Berezinskii-Kosterlitz-Thouless transition is manifested by a characteristic behavior of correlation functions [10]. In this paper we concentrate on a quasi-one-dimensional system, i.e. the situation when both: the chemical potential  $\mu$  as well as the thermal energy  $k_B T$  are smaller than a transverse confinement energy  $\mu, k_B T \lesssim \hbar \omega_r$ , where  $\omega_r$  is the transverse trap frequency.

One-dimensional systems are different than their 2D or 3D analogues. First of all there is no phase transition from the thermal to the condensate phase both in a trap and a uniform system. There exists however a degeneracy temperature  $T_d = \hbar^2 n^2 / 2m$  ( $n$  is a 1D density) below which the quantum effects become important in the uniform gas at the thermodynamic limit. Similarly, for the ideal 1D gas confined in the harmonic trap of frequency  $\omega$ , the characteristic temperature of quantum degeneracy is equal to  $T_c = (\hbar\omega/k_B)N/\log(2N)$ . If, in addition to a harmonic confinement, the gas is weakly interacting, then there exist a second characteristic temperature  $T_\phi = 15(\hbar\omega_z)^2 N / 32\mu < T_c$ . As shown in [11] for temperatures range in between the two  $T_\phi < T < T_c$ , the system is in a quasicondensate regime where density fluctuations are suppressed but phase fluctuations are large. Only below  $T_\phi$  the system enters a fully coherent region with suppressed density and phase fluctuations. The phase fluctuations have been observed in expansion when they are transformed into density fluctuation [12]. The statement of suppressed density fluctuations remains a common believe as there is no direct experimental evidence supporting or disclosing the above mentioned theoretical prediction based on the Bogoliubov approach.

For a very elongated quasi-one-dimensional system the local density approximation is justified. Then both: the trapped and the uniform systems should exhibit similar properties. The uniform system interacting via the zero-range pseudo-potential of the strength  $g$  is formally exactly soluble via the Bethe ansatz. In the limit of strong interaction, i.e. if  $\gamma = mg/\hbar^2 n \gg 1$  the system reaches the Tonks-Girardeau limit of impenetrable bosons i.e. the regime of fermionization. The system we investigate here is in the opposite regime of parameters, namely the regime where interactions are small.

One-dimensional Bose gases are special systems in a sense that, as discovered many years ago by Lieb [13], they exhibit two families of elementary excitations. As already identified by Lieb [13], the main branch is related to phonons. It turns out that dark solitons can be associated to the type II branch of the elementary excitations. That has been demonstrated by analyzing the dispersion relation for solitary waves [14] and also by studies of quantum nature of dark solitons beyond the mean-field [15]. Recently, we have shown that dark solitons are spontaneously generated in quasi-one-dimensional Bose gases at equilibrium [16]. By analyzing the statistical distributions of excitations within both the Lieb-Liniger model [17] and the classical fields approximation [18, 19] we proved that type II excitations are indeed quantum solitons [20].

The spectrum of elementary excitations plays a crucial role for the correlation functions and coherence of the system. There is a reach literature devoted to both the first order [21, 22] and the second order correlations [23–26] of the 1D or quasi-one-dimensional systems. A number of different approximate formulas for the second order correlation function in various regimes of the sys-

tem parameters and temperatures are available. In particular in the region of fermionization the antibunching of atoms is predicted. On contrary, at temperatures close to the degeneracy temperature and in the limit of weak interactions, the bunching of atoms is expected. The piece-wise valid formulas, although quite interesting, do not give a simple and clear picture of the density-density correlations in the entire range of temperatures across the ‘BEC-like transition’ for a weakly interacting system where the Bogoliubov theory adopted to elongated system can be compared to the predictions based on the Lieb-Liniger model.

Observation of a spatial dependence of the second order correlation functions is difficult for several reasons. First of all observations in situ are limited by a finite resolution of detectors and, in addition, the standard detection techniques involve a partial spatial averaging – therefore a column density is monitored only. The most common method of detection is destructive and the atomic cloud is observed after releasing atoms from the trap followed by the expansion. All these effects smear out most of density fluctuations. Earlier studies of the second order correlation function in expansion considered ballistic expansion of noninteracting atomic cloud [27] showing that the correlation function observed is related to the correlation in situ by some scaling only. In [28] the authors showed that density-density correlations of the interacting uniform gas manifest themselves in the expansion in a form of density ripples. The spectrum of the ripples is related to density-density correlations. The recent experimental results of [29] for the two-point density correlation measured across the Bose-Einstein condensation temperature in elongated expanding system are still awaiting full understanding and theoretical description. The authors support the experimental results with the ideal gas theory and evidently one should account for interactions to get a deeper insight into measured density correlations.

In this paper we are going to demonstrate that the properties of the density correlations of a Bose gas at temperatures below the critical one, after the gas is released from the trap, indicate large density fluctuations in the quasicondensate regime and might be a signature of the presence of thermal solitons in the system. As our calculations show, for elongated Bose gases the density correlation function takes large values below the threshold temperature, i.e. for  $T < T_c$ . These values are larger than those calculated for a weakly-interacting Bose gas within the Bogoliubov approximation and differ even more significantly from the results obtained for the ideal Bose gas theory. This indicates that not only phonons but also the second branch of elementary excitations influences the properties of density correlations. Large values of the density correlation function are therefore a signature of the presence of dark solitons. This should not be very surprising as an every dark soliton is associated with a density dip. The recent experiment showing large values of the density correlation function

for temperatures below the critical one [29] (see Fig. 3c) support our finding.

Below we are going to describe our numerical approach to determine the density-density correlation function of the elongated interacting Bose gas after expansion from the harmonic trap, taking into account a realistic detection procedure, in particular a column averaging and finite resolution of a CCD camera.

In second quantization formalism one introduces the field operator  $\hat{\Psi}(\mathbf{r})$  ( $\hat{\Psi}^\dagger(\mathbf{r})$ ) which annihilates (creates) an atom at position  $\mathbf{r}$ . The second order correlation function,  $G^{(2)}(\mathbf{r}, \mathbf{r}')$ , is then defined as a statistical average

$$G^{(2)}(\mathbf{r}, \mathbf{r}') = \langle \hat{\Psi}^\dagger(\mathbf{r}) \hat{\Psi}(\mathbf{r}) \hat{\Psi}^\dagger(\mathbf{r}') \hat{\Psi}(\mathbf{r}') \rangle. \quad (1)$$

It is convenient to use a normalized second order correlation function

$$g^{(2)}(\mathbf{r}, \mathbf{r}') = \frac{G^{(2)}(\mathbf{r}, \mathbf{r}')}{G^{(1)}(\mathbf{r}, \mathbf{r}) G^{(1)}(\mathbf{r}', \mathbf{r}')}, \quad (2)$$

where  $G^{(1)}(\mathbf{r}, \mathbf{r}') = \langle \hat{\Psi}^\dagger(\mathbf{r}) \hat{\Psi}(\mathbf{r}') \rangle$  is the first order correlation function.

To calculate the second order correlation function we turn to the approximate treatment of an interacting Bose gas, which is called the classical fields method (CFA) [18, 19]. The idea of the approach can be considered as an extension of the original Bogoliubov idea [30], i.e. the bosonic field operator  $\hat{\Psi}(\mathbf{r})$  is replaced by the complex wave function  $\Psi(\mathbf{r})$ . This wave function corresponds to a state having large energy related to the temperature and is composed of macroscopically occupied single particle modes only. To obtain the thermal equilibrium state of an elongated weakly interacting Bose gas we generate an ensemble of classical fields  $\Psi(\mathbf{r})$  corresponding to a given temperature. An effective way of getting members of such a canonical ensemble is to use the Monte Carlo algorithm [31].

Each classical field belonging to the canonical ensemble obeys the following equation of motion [19]:

$$i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = \left[ -\frac{\hbar^2}{2m} \nabla^2 + V_{\text{trap}}(\mathbf{r}, t) \right] \Psi(\mathbf{r}, t) + g \Psi^*(\mathbf{r}, t) \Psi(\mathbf{r}, t) \Psi(\mathbf{r}, t), \quad (3)$$

where the coupling constant  $g$  describes the contact interaction between atoms. It looks like the usual Gross-Pitaevskii equation for the Bose-Einstein condensate at zero temperature. However, here the complex function  $\Psi(\mathbf{r})$  carries information on both the condensed and non condensed atoms. The condensate and the thermal cloud can be split via the coarse-graining procedure [19, 32]. In short, the coarse-graining allows to separate the mode characterized by the largest first order coherence length (or a coherence time) from the remaining part of the classical field.

Different realizations of classical fields, being the members of the canonical ensemble, represent the set of

atomic clouds at the thermal equilibrium. While evolving with the help of Eq. (3) the classical field reveals the properties of an atomic cloud within a single-shot experiment. We therefore pick up typically thousands of members of the canonical ensemble and propagate them without an external trapping potential, i.e.  $V_{\text{trap}} = 0$ , for a given time. We then compute the autocorrelation function of the density integrated along one of the radial directions  $\tilde{n}_{2D}(x, z) = \int |\Psi(x, y, z)|^2 dy$ , where  $z$  is the axial direction. However, because of the finite resolution of the optical imaging system we smooth the radially integrated density via the following convolution:

$$n_{2D}(\mathbf{r}) = A \int \tilde{n}_{2D}(\mathbf{u}) \exp[-(\mathbf{r} - \mathbf{u})^2 / 2\sigma^2] d^2u, \quad (4)$$

with the Gaussian function of the half width at half maximum (HWHM) of about a few micrometers and  $A = 1/2\pi\sigma^2$ . The density correlation function itself is obtained by averaging the autocorrelation  $\int n_{2D}(\mathbf{u}) n_{2D}(\mathbf{u} + \mathbf{r}) d^2u$  over all performed realizations:

$$G^{(2)}(0, \mathbf{r}) = \langle \int n_{2D}(\mathbf{u}) n_{2D}(\mathbf{u} + \mathbf{r}) d^2u \rangle. \quad (5)$$

To get the normalized density correlation function we divide (5) by the autocorrelation function of the mean density [33], integrated along the radial direction:

$$g^{(2)}(0, \mathbf{r}) = G^{(2)}(0, \mathbf{r}) / \int \langle n_{2D}(\mathbf{u}) \rangle \langle n_{2D}(\mathbf{u} + \mathbf{r}) \rangle d^2u. \quad (6)$$

Autocorrelation functions appearing in both the numerator and denominator of the above definition (6) are calculated using the fast Fourier transform.

Because we use the classical fields approximation, i.e. we replace the bosonic fields operators by the complex functions, the shot-noise term does not appear in our approach. The origin of the shot-noise term is in the commutation relations between the creation and annihilation operators for the bosonic field. Obviously in the classical fields approximation the bosonic commutation relations are not accounted for. It might be questionable to use the CFA for obtaining the density correlation function because the shot-noise term appears explicitly in the expression for the second order correlation function while expressed with the help of the first order correlation functions. The shot-noise term is of the order of the number of particles in the system,  $N$ , whereas the other terms are proportional to  $N^2$ . Hence, it can be neglected for large systems. Although, it is detectable in the experiment of Ref. [29], a special care is taken to exclude the atomic shot-noise peak from the experimental data.

In Fig. 1 we show the density correlation function,  $g^{(2)}(x, z)$ , for the temperature  $T = 3.15$  nK after free expansion over 46 ms (upper and lower-left frames). The Bose gas consisting of  $10^3$  rubidium atoms is initially confined in a quasi-one-dimensional harmonic trap with the radial frequency  $\omega_r = 2\pi \times 113$  Hz and the axial one  $\omega_z = 2\pi \times 1$  Hz. The aspect ratio is just as for the most

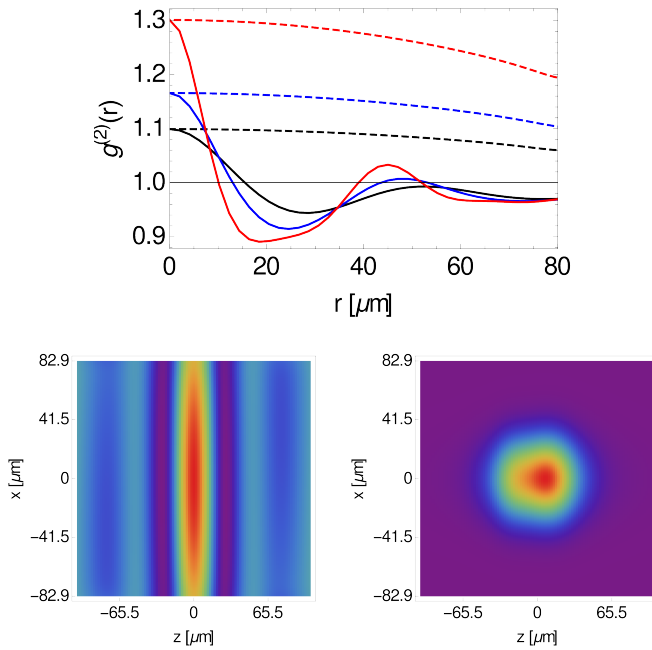


FIG. 1: (color online). Axial (solid lines) and radial (dashed lines) cuts of the density correlation function  $g^{(2)}(x, z)$  of the Bose gas after 46 ms of expansion (upper frame). The Bose gas consisting of  $10^3$  rubidium atoms is initially confined in a harmonic trap with radial and axial frequencies  $\omega_r = 2\pi \times 113$  Hz and  $\omega_z = 2\pi \times 1$  Hz, respectively. The system is prepared in the thermal equilibrium at the temperature  $T = 3.15$  nK. Three sets of curves are obtained by taking different values of the smoothing parameter:  $\sigma = 4 \mu\text{m}$ ,  $6 \mu\text{m}$ , and  $8 \mu\text{m}$  (from top to bottom). Lower panel: The density correlation function  $g^{(2)}(x, z)$  for  $\sigma = 8 \mu\text{m}$  (left frame) and the two-dimensional density (i.e., density averaged along the direction of imaging) after expansion, first smoothed with the parameter  $\sigma = 8 \mu\text{m}$  and next averaged over all realizations (right frame).

elongated trap used in the experiment of Ref. [29]. Similarly to the experiment, the theoretical system is also in the quasi-one-dimensional limit, i.e. its chemical potential is smaller than the radial excitation energy,  $\hbar\omega_r$ . Some other parameters are different, in particular the number of atoms is 10 times smaller. We use the system with smaller number of atoms confined in a trap with lower trap frequencies to decrease its mean field energy. In such a case the expansion of the gas after the trap is removed is much slower and becomes feasible numerically. We believe, however, that the scaling does not alter significantly the main physical processes responsible for behavior of the density correlation function.

The gas is then released from the trap and expanded for 46 ms. For that we solve the Eq. (3) with no trapping potential,  $V_{\text{trap}} = 0$ . The expansion is not rapid and after 46 ms the radial size of the cloud remains at the order of tens micrometers (see lower-right frame in Fig. 1, where we show the density, first smoothed with the parameter  $\sigma = 8 \mu\text{m}$  and then averaged over all realizations). As op-

posed to the experiment of Ref. [29] in our calculations we image the whole radial extension of the cloud. The upper frame in Fig. 1 presents the density correlation function cuts along the axial (solid curves) and radial (dashed curves) directions. The bunching at short inter-particle distance ( $g^{(2)} > 1$ ) as well as the oscillations of correlations along axial direction with the values below one are clearly visible. Our results correspond to the observations in the experiment of Ref. [29] (see also earlier experimental data of Ref. [33]). At large distances which coincide with the edge of the atomic cloud, the density correlation function reaches its limit equal to one.

Speaking more precisely, during the expansion, in the limit of large expansion time, we monitor the momentum distribution, i.e., the Fourier transform of the classical field. We find that it does not change in time almost at all what means that the interaction energy does not play any significant role during the expansion in our case, i.e., for the system with  $N = 1000$  atoms. In practice the classical field evolves freely and at any time can be found with the help of the propagator of the free Schrödinger equation. This technique is much faster than direct solving of the Eq. (3) on the grid and, as we checked, gives the same results. In fact, this technique was used by us since while calculating the density correlation function, the Eq. (6) has to be averaged over a few thousand of realizations.

Next, we analyze the peak value of the second order correlation function,  $g^{(2)}(0, 0)$ , of expanding Bose gas for various temperatures. There is a huge discrepancy between the experimentally measured values and the results predicted by the ideal Bose gas theory for elongated systems, see Ref. [29]. For temperatures below the critical one the peak height of  $g^{(2)}(x, z)$  rapidly decreases to unity for the ideal gas case whereas experiment proves that the peak height does not change significantly within a large range of temperatures. In Fig. 2 we mark the results obtained within the classical field approximation by red dots. The temperature is given in units of the critical temperature for the ideal Bose gas corrected due to finite number of atoms and the quasi-one-dimensional character of the trapping potential. For the ideal Bose gas in the thermodynamic limit the critical temperature is  $T_c = \hbar\bar{\omega}N^{1/3}/k_B [\zeta(3)]^{1/3}$ , where  $\bar{\omega} = (\omega_r^2\omega_z)^{1/3}$  and  $\zeta()$  is the Riemann zeta function [34]. For the parameters we use in our calculations,  $T_c = 10.5$  nK. After corrections mentioned above the critical temperature is, however, shifted down and equals  $T_c = 6.3$  nK [34]. Since in the system we consider the number of atoms is very small, the interactions do not influence the critical temperature much.

Now we compare our results with the ones obtained within the theoretical model described in Ref. [28]. This model allows to calculate the two-point density correlation function of very elongated ultracold Bose gas after its release from the trap. For a tight radial confinement the transverse atomic motion is essentially frozen and thus decouples from the motion along the long axis.

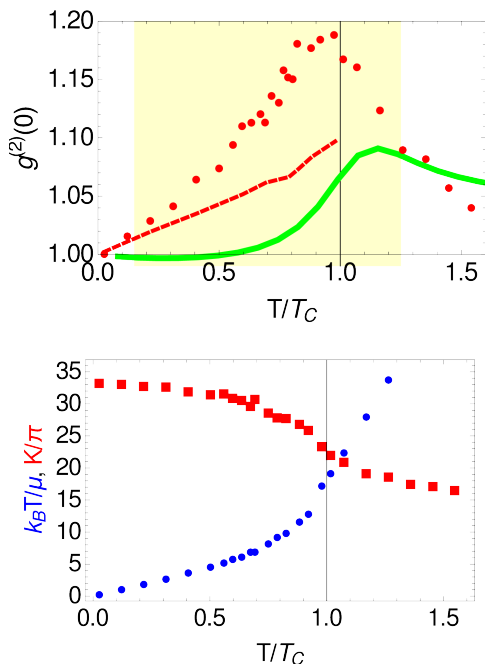


FIG. 2: (color online). Upper frame: Peak height of the density correlation function as calculated within the classical field approximation (red dots), the ideal Bose gas model as in Ref. [27] (green solid line), and the Bogoliubov approximation extended to low-dimensional quasicondensates as in [28] (red dashed line). The critical temperature equals  $T_c = 6.3$  nK and the smoothing parameter  $\sigma = 8$   $\mu\text{m}$ . The shaded area shows the range of temperatures when deep spontaneous dark solitons appear in the system. Note large values of the density correlation function below the threshold temperature calculated within the classical field approximation. Lower frame:  $k_B T / \mu$  (blue dots) and  $K / \pi$  (red squares) as functions of temperature to confront the validity of the model of Ref. [28] (red dashed line in the upper frame).

Therefore, the problem of calculating the density correlations is effectively reduced to 1D. The density correlation function turns out to be at any time analytically related to the spectrum of “density ripples” which, on the other hand, can be obtained within the Bogoliubov approximation extended to low-dimensional quasicondensates. The shape of the density correlation function depends on the expansion time, temperature, and the 1D density. The 1D density is calculated as the average of the radially integrated central densities of classical fields belonging to the canonical ensemble corresponding to the given temperature. The red dashed line in Fig. 2, upper frame, is the output of such calculations. One has to remember, however, that the validity of Ref. [28]’s model is restricted to such temperatures that  $k_B T / \mu \ll K / \pi$ , where  $K = \pi \hbar n_{1D} / (mc)$  is the Luttinger liquid parameter with  $c$  being the speed of sound. In Fig. 2, lower frame, we show both quantities,  $k_B T / \mu$  and  $K / \pi$ , to visualize the above mentioned condition. Certainly, the results of this model are questionable for temperatures about the critical one. On the other hand, for lower temperatures it is

clear that the interactions strongly influence the density correlation function.

Fig. 2 clearly demonstrates the differences between theoretical predictions based on the ideal Bose gas model as well as on the model of an interacting gas within the Bogoliubov approximation and the ones obtained in the framework of the classical fields approximation. Only CFA results qualitatively recover the experimental data of Ref. [29] showing the significant density correlations below the critical temperature for the elongated Bose gases. The reason is that CFA reproduces the correct spectrum of elementary excitations including not only the Bogoliubov phonons but also the II type excitations – the dark solitons. They do occur spontaneously in a quasi-one-dimensional interacting Bose gas at equilibrium as it was already reported in [16]. The dark solitons do not affect the first order correlation function, i.e. the coherence length as shown in [16], however they affect the density fluctuations. Large peak values of density correlations can be associated with the existence of solitons, see Fig. 3. As it was already shown many years ago by Lieb [13, 17] and later on generalized to the finite temperature case by Yang and Yang [35], a one-dimensional system exhibits two families of excitations. The one branch belongs to phonons and is well described by the Bogoliubov approximation. This kind of excitations are included in the model of Ref. [28]. However, there is the second branch of excitations which are dark solitons [20] which is neglected in the considerations in Ref. [28]. Hence, the origin of the discrepancy between the two approaches is, in our opinion, related to the presence of the second type of excitations in one-dimensional systems, i.e. to the dark solitons. They are especially important in the regime of a quasicondensate where very deep solitons are possible due to large thermal energy.

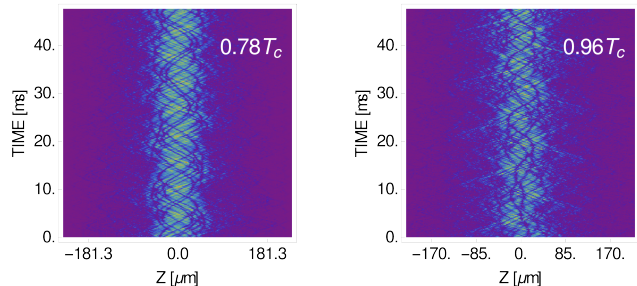


FIG. 3: Solitons for trapped atomic clouds at thermal equilibrium for the temperatures  $T = 4.94$  nK and  $T = 6.09$  nK. The panels show the time evolution of the density along the axial direction. Dark solitons are clearly visible as dark lines winding from the one edge of atomic cloud to the other.

Above the critical temperature the peak density correlation should be equal to two, indicating atom bunching. The CFA calculations, however, give a value of the second order correlation function close to one rather than two. This is related to the fact that the characteristic length over which the particles bunching vanishes

(i.e.  $g^{(2)}(0) = 1$ ) decreases with increasing temperature. Eventually, this length becomes smaller than the spatial resolution of the imaging system and the bunching effect cannot be visible any more. Our calculations take into account the realistic resolution of detectors, therefore they give a decrease of the correlations above the critical temperature as it is observed in the experiment [29]. In our case, the decrease of  $g^{(2)}(0)$  is slower since we have fewer number of atoms in the system.

In summary, we have studied the density correlation function of expanding Bose gas. The weakly interacting Bose gas at thermal equilibrium is confined initially in a very elongated trap. The whole range of temperatures is considered, from temperatures as low as those of a pure condensate through the ones typical for quasi-condensates up to temperatures much above the critical one. Below the critical temperature the normalized density correlation function does not fall rapidly to the value one as expected for the coherent system and predicted for the ideal Bose gas [27]. Our finding sheds a new light onto the understanding of coherence of elongated or quasi-one-dimensional system. Contrary to what was expected, relatively large phase and density fluctuations signify a lack of a full first and second order coherence

and atom bunching in a large range of temperatures below the degeneracy temperature. Indeed, the interactions are responsible for these features. However, the theory based on the Bogoliubov approximation alone seems not to be able to fix the problem. This is because in elongated systems besides the phonon excitations described by the Bogoliubov approximation there exist another excitations as predicted by the Lieb-Liniger model. These are the dark solitons. Our approach captures both kinds of excitations and therefore qualitatively recovers the experimental data of Ref. [29].

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